Stability of modified electroweak strings

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Abstract

We discuss the stability of an electroweak string with axions in its core, which give to the configuration a quasi-topological property, and compare it with other modifications using instantons in the thin-wall approximation.

1 Introduction

The existence of electroweak strings might be of relevance both for the explanation of cosmological problems and future accelerator results [1]. However, when they are built exclusively with Higgs and gauge fields of the standard model we obtain classically unstable configurations, or at most quantum metastable ones with significant life-time only for too small Higgs boson mass [2] and too large Weinberg angle [3].

To give more stability to the electroweak string, an additional global abelian symmetry may be considered. When this symmetry is unbroken, its Noether charge for particles lighter inside than outside the core may produce stable strings as occurs for non-topological solitons [4]. Alternatively, if the global symmetry is broken, there is a topological reason for the stability of a configuration where the electroweak component is added to an axionic string [5], though QCD effects may cause its decay. We propose a closed string, that partially combines both effects, with axions in its core which are lighter than outside and a quasitopological basis for stability due to the phase nature of the axion field that allows a variation of a multiple of 2π along it. Absolute stability is not obtained due both to the very small decay probability of axions and to the finiteness of the transverse region of this coherent configuration inside the core.

In section 2 we describe the reasons of instability of electroweak strings and attempts to stabilize them. Section 3 is devoted to the proposed string with axions in its core showing that the configurations might be macroscopic and with large life-time. Brief conclusions are included in section 4.

2 Electroweak strings and attempts of stabilization

We remind that the stability of vortices relies on the non-triviality of $\Pi_1(M)$, where M is the vacuum manifold, and eventually on the conservation of the magnetic flux through the core in the gauge theories. In the case of a broken global $U(1)$ symmetry, the stability of the so called axionic string is assured by $\Pi_1(U(1)) = Z$. For the Nielsen-Olesen vortex [6] corresponding to the breaking of a local $U(1)$ symmetry, if this $U(1)$ was the remnant of a larger broken symmetry G such that $\Pi_2(G/U(1)) \neq I$ a quantum decay of the

configuration to a broad one may occur[7].

The limit $g = 0$ of the electroweak string is the semilocal one [8] which corresponds to the breaking $SU(2)_{gl} \times U(1)_{loc} \rightarrow U(1)_{gl}$ with a complex Higgs field $\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$ where φ_2 is electrically neutral, a potential

$$V = \frac{\lambda}{4} \left(|\varphi_1|^2 + |\varphi_2|^2 - v^2 \right)^2 \tag{1}$$

and a neutral gauge field Z with coupling g'. Though in this case $\Pi_1(M) = I$, the conservation of Z-magnetic flux may produce stability of the vortex. In fact being the configuration such that for large r in $D = 2$

$$\varphi \rightarrow \begin{pmatrix} 0 \\ v \end{pmatrix} e^{i\theta}, \quad Z_\theta \rightarrow \frac{1}{r} \tag{2}$$

and for small r $\quad \varphi \rightarrow 0, \quad Z \rightarrow 0$, if we increase φ_1 up to v inside the core and then enlarge its size $R \rightarrow \infty$, the remaining energy is the additional scale invariant gradient contribution $\sim v^2$. The vortex will be stable if its potential and magnetic energies $\sim \sqrt{\lambda} v^2 / g' = \frac{M_H}{M_Z} v^2$ are smaller than the previous gradient contribution, being this possible if the Higgs mass is lighter than the vector boson one[7]. If on the contrary $M_H > M_Z$ the vortex becomes metastable tunneling to a broader configuration with the same energy. Finally, for a sufficiently large $M_H \sim 2M_Z$ the radius of the latter configuration becomes equal to the original vortex size, there is no barrier to surmount and the defect is unstable. A channel for decay of the $D = 3$ string is its breaking by nucleation of a global monopole pair. Again for large $M_H \sim 2M_Z$ the string will become unstable when the energy balances for a pair separation of the order of the defect size so that there will be no barrier to surmount.

In the true electroweak case $g \neq 0$, $\tan \theta = g'/g$, the B_Z-flux is not conserved and may transform into the electromagnetic B_A-flux [9]. Therefore the vortex can never be stable because one may change the configuration eliminating the phase of φ_2 in Eq.(2) by a gauge transformation, then turning B_Z-flux into B_A-flux which is decoupled from φ_2 thus avoiding a divergent energy contribution when φ_2 is subsequently increased to v inside the core. No gradient contribution is introduced in this way and enlarging $R \rightarrow \infty$ to eliminate the magnetic energy, one obtains a vacuum configuration of zero energy without having surmounted any infinite barrier. For $D = 3$ one decay channel is given by the nucleation of magnetic monopoles with the probability P given by the bounce action B [7]

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$$P \sim e^{-B}, \quad B \sim \frac{E_{mon}^2}{\mu_s} \tag{3}$$

Analogously to the semilocal case, the string energy per unit length is $\mu_s \sim \sqrt{\lambda} \sin \theta v^2/g' = \frac{M_H}{M_Z}v^2$ and the monopole energy for the realistic case $g > g'$ can be estimated by the sum of the contribution of a global monopole for a small range $< 1/M_Z$ and that of the magnetic energy emerging from it i.e.

$$E_{mon} \simeq \frac{v^2}{M_Z} + \frac{\sin^2\theta}{g^2} M_Z = \frac{M_Z}{g^2} \tag{4}$$

The energy balances for a separation of monopoles which is larger than their size, condition for metastability, for large θ and small $\frac{M_H}{M_Z}$ excluding therefore the realistic region. The same conclusion is reached for $D = 2$ analizing the monopole pair bounce configuration responsible for the decay of the Z-string into a B_A-flux configuration. It is important to remark that the instability of Z-strings is additionally increased by its possible decay into a W^+W^- condensate [10]. This is due to the vacuum instability caused by the anomalous coupling of the spin of W with A and Z magnetic fields [11]. In particular Ws are produced with a zero energy expense for a strong Z magnetic field

$$B_Z \gtrsim \frac{M_W^2}{g\cos\theta} \tag{5}$$

which, for the flux inside a Z-string, is satisfied for $M_H \gtrsim M_Z$.

To obtain a larger stability one may think of including an additional global symmetry $\tilde{U}(1)$. If this symmetry is unbroken there is no topological reason for stability since one still has $\Pi_1(M) = I$ but the conservation of its Noether charge may stabilize the configuration as occurs in non-topological solitons. Thus an example is to introduce a complex scalar field χ with the additional lagrangian [4]

$$\mathcal{L}_\chi = |\partial_\mu \chi|^2 - \lambda_2 |\chi|^4 - \lambda_3 \left(\varphi^\dagger \varphi - m_0^2 \right) |\chi|^2, \quad v^2 > m_0^2 \tag{6}$$

with parameters such that the mass of the particle χ is smaller inside the vortex core, where there is an expectation value χ_0, than its mass $m_\chi = \sqrt{v^2 - m_0^2}$ outside. For $D = 2$ we may pass from the vortex configuration in whose core $\varphi \sim 0$, $|\chi| \simeq \chi_0$ to the vacuum $\varphi_2 = v$, $\chi = 0$ everywhere

by the same steps as in the electroweak case with the only difference that the vortex energy decreases from $\frac{M_H}{M_Z}v^2$ to $\frac{\sqrt{M_H^2 - \delta/v^2}}{M_Z}v^2$, where δ corresponds to the depth of the local minimum of the potential caused by Eq.(6). The replacement of the Higgs mass by $\sqrt{M_H^2 - \delta/v^2}$ in all the above considerations tends to increase the life-time of the metastable configurations and to allow them for more realistic values of M_H. For $D = 3$ it would be possible to obtain absolute stability if we consider z, t dependent configurations inside the core corresponding to a Noether charge Q per unit length. If the particles χ are so heavy outside the core that $Qm_\chi > \tilde{\mu}$, where $\tilde{\mu}$ is the energy per unit string length including the Q particles inside it, the string cannot decay. Otherwise for the monopole nucleation channel the bounce action of Eq.(3) is modified through the replacement of μ by $\tilde{\mu} - Qm_\chi$ with the possibility of increasing the life-time.

It is interesting to consider the realistic case in which the fermions that acquire their mass through coupling with the Higgs field may be trapped in the string core. It has been shown that in this way the electroweak string becomes superconducting [12] with zero-mass fermions running along one direction of the string or the other according to their being an up or a down component of the elecroweak doublet [13]. But the Dirac sea of these quarks tends to destabilize the string [14] because a small perturbation of the upper Higgs component $\delta\varphi_1$ produces, via the Yukawa coupling h to the fermion, mixed states for each momentum p with an energy shift

$$\Delta\epsilon = -\left(\sqrt{p^2 + |h\delta\varphi_1|^2} - p\right) \tag{7}$$

It is therefore necessary that positive energy fermions with a proper density per unit string are present to overcome this shift and may have a chance to stabilize the configuration.

One may think to increase the stability to the elecroweak string introducing topological reasons for it. A possibility is to make the hypothetical axion field play a role due to its phase nature. A model is built adding the electroweak components to the axionic string [5] which corresponds to a global $\tilde{U}(1)$ symmetry spontaneously broken at a higher scale f_{PQ} so that

$$\Pi_1\left(SU(2) \times U(1) \times \tilde{U}(1)/U(1)_{em}\right) = \Pi_1\left(\tilde{U}(1)\right) \neq I.$$

To have a coupling between the phase of the scalar field ψ which breaks this Peccei-Quinn symmetry with the Higgs sector one needs two complex Higgs

doublets φ_i and an effective potential

$$V(\varphi_i, \psi) = \sum_{i=1}^{2} \frac{\lambda_i}{4} \left(\varphi_i^\dagger \varphi_i - v_i^2\right)^2 + \frac{\lambda}{2} \left(\varphi_1^\dagger \varphi_1\right) \left(\varphi_2^\dagger \varphi_2\right) +$$
$$+ \frac{\lambda'}{2} \left(\varphi_1^\dagger \varphi_2\right) \left(\varphi_2^\dagger \varphi_1\right) + f_{PQ} v_1 v_2 - \frac{1}{2} \left[\left(\varphi_1^\dagger \varphi_2\right) \psi + h.c.\right]$$

(8)

Eq.(8) forces the difference of phases of the Higgs doublets to be equal to that of ψ outside the string core i.e. $\theta_1 - \theta_2 = \theta_\psi$ with

$$\varphi_i = \begin{pmatrix} 0 \\ v_i \end{pmatrix} e^{i\theta_i}, \quad \psi = f_{PQ} e^{i\theta_\psi} \tag{9}$$

A modification emerges from QCD corrections which give a small mass m_a to the axion breaking intrinsically the symmetry $\tilde{U}(1) \to Z(N)$, where in the invisible axion model N is the number of quark flavours. Therefore the string becomes attached to N axionic walls which have too much energy for cosmology [15]. If the attached wall is a single one it becomes surrounded by the string turning into a metastable configuration [7]. When the electroweak components are included into the axionic string, the Z-flux is conserved due to Eq.(9) adding stability to the configuration because its electroweak part returns to the situation of the semilocal case.

3   Electroweak string with axions in its core

We propose a different mechanism for the stabilization of the electroweak string, inspired in a previously discussed non topological configuration [16], which consists in including axions in the core combining the effects of being lighter there and of corresponding to a phase-like field. In the case analyzed above one thinks that the skeleton is an axionic string formed at the Peccei-Quinn scale $f_{PQ} \sim 10^{12} \, GeV$, which attracts electroweak components at the $SU(2) \times U(1)$ breaking. We consider instead that the electroweak strings formed at the latter scale trap axions. While outside the core the real scalar axion field must be around zero in correspondence to the vacuum, it takes an average value a_0 inside the core where the electroweak symmetry is unbroken. This follows from the absence of the contribution $\arg \det \mathcal{M}$ of the mass matrix [17] so that it must not be compensated by the axion field.

To explain this feature we remind that the vacuum energy $E(\theta)$ of QCD in a phase where the quarks have mass m depends on the $\bar\theta$-vacuum parameter as [18]

$$\exp - \left(E(\bar\theta)VT\right) = \int \mathcal{D}G_\mu \det(\not{D} + m) \\ \cdot \exp - \int d^4x \left(\tfrac{1}{2}G_{\mu\nu}G_{\mu\nu} - i\bar\theta G_{\mu\nu}\tilde{G}_{\mu\nu}\right) \tag{10}$$

with $G_{\mu\nu}$ gluon fields.

Since $\det(\not{D}+m) = m^{N_0}\sum_r (\lambda_r^2 + m^2) > 0$ the factor $e^{i\bar\theta\nu}$ can only decrease the integral so that $E(\bar\theta) > E(0)$. In case that the axion field is included, all what said above applies to $\bar\theta - a'/f_{PQ}$ which is defined as $-a/f_{PQ}$. To be precise, $\bar\theta$ contains not only the QCD parameter θ but also $\arg\det\mathcal{M}$ coming from the EW breaking.

If we are in the symmetric EW phase and $\det\not{D}$ is still positive a' must compensate only θ and the redefined a will be of order f_{PQ}. If the axion mass in this phase is vanishing in correspondence to zero modes of $\not{D}$ there will not be a preferred value of a but the average of the phase a/f_{PQ} will be of order 1.

What said above may be simulated in the simplest way by an effective potential

$$V(\varphi, a) = \frac{\lambda}{4}\left(\varphi^\dagger\varphi - v^2\right)^2 + \left[m_a^2 f_{PQ}^2 + \kappa\left(\varphi^\dagger\varphi - v^2\right)\right]\left(1 - \cos\frac{a}{f_{PQ}}\right) \tag{11}$$

with $\kappa v^2 > m_a^2 f_{PQ}^2$. In the stable phase $|\varphi| = v$ Eq.(11) gives the normal axion potential with mass m_a which forces the field to its minimum $a = 0$. Outside the string we expect this phase and inside it, where $\varphi \sim 0$, the axion mass will be

$$m_a'^2 = \kappa\frac{v^2}{f_{PQ}^2} - m_a^2 \tag{12}$$

which may be adjusted to the desired extremely small value by the additional parameter κ.

We will now show that both decay channels of monopole nucleation and W condensate may be considerably suppressed by this mechanism.

For $D = 3$, if we make a vary along the axis of the string particles are excited and being $m_a' \ll m_a$, we obtain an effect equivalent to that of the Noether charge because of the very long life-time of axions. But more

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important than that, due to the fact that a/f_{PQ} is a phase, if we take a closed string the variation along it must be $\Delta a = n2\pi f_{PQ}$ with integer n, and there is a quasi-topological reason for an increase of the stability as it occurs in the superconducting strings [13]. This follows from the fact that when the string is cut an additional energy corresponding to a jump of a in the wall width ϵ must be supplied. The decay of the string must be accompanied by an alignment of the phase a of the field ψ inside the defect with its value $a = 0$ outside in order to minimize the gradient energy. This alignment may be done smoothly in all the points along the string axis except for a small region where an abrupt change of phase will appear which will require to cut the string (see Fig.1) with a gradient energy of order

$$d \sim \left(\frac{\pi f_{PQ}}{\epsilon}\right)^2 \epsilon \pi R^2 \tag{13}$$

that is not infinite because of the finiteness of the transverse radius R .

It is important to note that the pure EW string has an energy per unit length which comes from minimizing essentially the contribution of the difference of vacua and that of the flux Φ, i.e.

$$\mu \simeq \Delta V \pi R^2 + \frac{\Phi^2}{2\pi R^2} \tag{14}$$

which leads to a field $\vec{B}_Z$ that is independent of R and exceeds the critical value Eq.(5) for $M_H > M_Z$ producing the instability of the configuration.

Now, the addition of axions inside the core gives a large surface contribution which together with the flux term determines the energy per unit length

$$\tilde{\mu} \simeq \frac{(\Delta a)^2}{\epsilon} 2\pi R + \frac{\Phi^2}{2\pi R^2} \tag{15}$$

Minimizing Eq.(15) $R^3 \sim \epsilon \Phi^2 / (\Delta a)^2$, taking the wall width $\epsilon \sim 1/\sqrt{m_a M_H} \sim 10^6 GeV^{-1}$ and being $\Delta a \sim f_{PQ}$, it is necessary that $\Phi \gtrsim 10^{18}$ to ensure $R \gtrsim \epsilon$. Even though this flux is a large number, if the radius is not too small $\vec{B}_Z$ may go below the critical value Eq.(5). In fact with $R \sim 10^4 \epsilon$, B_Z turns out to be $10^{24} G$. Therefore the string radius must be larger than $10^{-4} cm$ to avoid the instability due to W condensation.

Regarding the energy balance for the monopole pair nucleation which may break the string and including its cut contribution

$$2E_{mon} + d = \tilde{\mu}L \tag{16}$$

the bounce action is now

$$B \sim \frac{(2E_{mon} + d)^2}{\tilde{\mu}} \tag{17}$$

If we adopt the above parameters and taking the energy of the monopole as the magnetic contribution due to Φ outside a core of the size R, $E_{mon} \sim 10^{38} GeV$ which is of the same order of d coming from Eq.(13), whereas $\tilde{\mu}$ Eq.(15) is of order $10^{28} GeV^2$. Therefore $B \sim 10^{44}$ to be compared with the bounce action for a normal EW string $B \lesssim 1/g^4 \sim 10^2$ for $M_H \gtrsim M_Z$, *i.e.*, much more stable.

4   Conclusions

We have seen that at variance from other modifications a closed EW string with the inclusion of axion field in its core which increases continuously along it may exhibit a very long life-time, even though it is not completely stable. The high value of the energy of the string is exaggerated by the estimation of a common wall width for the Higgs and axion fields which gives a very large surface term for the latter. Therefore for quantitative applications a more accurate description of the configuration should be performed. It is clear that one should also study the mechanism for the production of such heavy and macroscopical objects which might play a role in cosmological aspects as the baryogenesis during the electroweak phase transition if this was weakly of first order or of second order [19] [20]. They might also influence astrophysical observations as a rotation of the polarization plane of the radiation from distant radiosources due to the parity-violating coupling with electromagnetic fields [21]. This effect caused by coherent axionic configurations may be more important than that due to the previously considered background of quasi-Goldstone boson fields [22].

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Figure caption

1a Closed string with axionic content. Arrows indicate axion phase changing in 2π inside the string whereas outside it is fixed to 0.

1b Cut of the string required to align the axion phase to its vacuum value.